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Papers submitted for publication should be addressed to the Editor and Manager, and should be accompanied with return postage.

The acceptability of papers offered must largely depend upon the judgments of our editorial referees. Final decisions will be made as promptly as possible and reported to those concerned.

This Journal is dedicated to the following aims:

1. THROUGH PUBLISHED STANDARD PAPERS ON THE CULTURE ASPECTS, HUMANISM AND HISTORY OF MATHEMATICS TO DEEPEN AND TO WIDEN PUBLIC INTEREST IN ITS VALUES.
2. TO SUPPLY AN ADDITIONAL MEDIUM FOR THE PUBLICATION OF EXPOSITORY MATHEMATICAL ARTICLES.
3. TO PROMOTE MORE SCIENTIFIC METHODS OF TEACHING MATHEMATICS.
4. TO PUBLISH AND TO DISTRIBUTE TO THE GROUPS MOST INTERESTED HIGH-CLASS PAPERS OF RESEARCH QUALITY REPRESENTING ALL MATHEMATICAL FIELDS.

Foreword

With this issue the title "Mathematics News Letter" becomes replaced by

"National Mathematics Magazine."

Apologies and explanations are not necessary. The older name, we believe, served its purpose well, but with the journal's expansion it was inevitable that an enlarging group of readers should regard "News Letter" as inaccurately descriptive.

The new title is a result of careful and deliberate thought participated in by all the members of our present Editorial Board, a plurality of whom favored the inclusion of "Magazine."

In keeping with a mechanical reformation whereby the new name and two new departments are initiated is the restatement of ideals and objectives. A study of the latter is recommended to readers.

The cleverly designed arrangement of the three words of the new title, with the accompanying symbolism, as shown on the cover, is the conception of Mrs. Irby C. Nichols, Baton Rouge, Louisiana. The monogram on the title page is also her creation.

Survey of Present Status of Secondary Mathematics in the United States

By S. T. Sanders

Late last spring the following questions were mailed from the News Letter editorial office to all state superintendents of education in the Union:

1. Has mathematics as a required subject for graduation from high schools been eliminated in your State?
2. If it has not been eliminated as a requirement is there a definite prospect that it will be eliminated at an early date?
3. In your judgement are there good reasons for the hope that mathematics will have increased use in the secondary schools of your State?

Replies have been returned from forty-six States and from the District of Columbia. They were furnished either by the superintendents themselves or by high school supervisors to whom in many cases the questions were referred.

It is our aim to present to the readers of *National Mathematics Magazine* the results of this survey and to do so in a form the most convenient and valuable. We believe that the latter object will be attained if we assemble all the materials belonging to each question under that question, assigning them to the proper States and the State officials returning the report.

In accord with this plan we present below all answers received to Question 1, namely:

"Has mathematics as a required subject for graduation from the high schools been eliminated in your State?"

ALABAMA

"Our state plan does not require mathematics for graduation from senior high school. In the junior high school mathematics is required in the first and second years, or seventh and eighth grades, and is offered in the ninth grade as an elective. Many schools are requiring that where the algebra of the ninth grade is not elected, some course in arithmetic or commercial work be required. However, that is not part of the state plan. The great majority of our schools are operating upon

this plan. There, are, however, some schools which will require both elementary algebra and geometry for graduation."—W. L. Spencer, Director of Secondary Education.

ARKANSAS

"The State does not list mathematics as one of the subjects required for graduation from high school in high schools which are in position to offer elective subjects in the academic field. Since the small high schools which cannot offer electives usually must meet college entrance requirements, a year and a half of Algebra and a year of Plane Geometry are included in the high school curriculum set up by the State."—M. R. Owens, State High School Supervisor.

ARIZONA

"Mathematics as a required subject for graduation from the high schools has not been eliminated in Arizona."—H. E. Hendrix, Superintendent of Public Instruction.

CALIFORNIA

"As a general requirement, yes. The state has liberalized its requirements for graduation and has made provision for the individual school district to set up its own standards, subject to the approval of the State Department of Education. Our program is based on the principle that every school system should have a very thorough-going guidance and counseling service and that every child should have developed for him a pattern of educational experience, and only those children who will need mathematics will be required to take it."—Vierling Kersey, Superintendent of Public Instruction.

CONNECTICUT

"In Connecticut there has been no general rule which required mathematics as an element in all high school courses which lead to graduation. It has, however, been a requirement in most of our towns and cities."—E. W. Butterfield, Commissioner of Education.

DELAWARE

"No."—H. V. Holloway, Superintendent of Public Instruction.

FLORIDA

"No."—W. S. Cawthan, State Superintendent of Public Instruction.

GEORGIA

"Mathematics is a required subject for graduation in Georgia high schools."—M. D. Collins, State Superintendent of Schools.

ILLINOIS

"Mathematics has never been required by this office for graduation from a recognized high school. Most high schools have been guided, however, by five fixed units of requirement set up by various colleges and universities. These units are as follows: English—three, Algebra—one, and Plane Geometry—one.

Last year, however, the University of Illinois puts its entrance requirements on a different basis and has eliminated this fixed list of subjects. Under the new requirements a candidate seeking entrance to the University must present at least three majors or two majors and two minors. It is possible for him to secure these majors and minors without having had any mathematics."—Harry M. Thrasher, State High School Supervisor.

INDIANA

"We require for graduation one unit of mathematics. However, practically all of our high school graduates do more than the one year of work in the mathematics."—V. R. Mullins, Director, School Inspection Division.

IOWA

"There is considerable sentiment in Iowa for the elimination of mathematics as a required subject for high school graduation."—R. C. Williams, Director of Research.

KANSAS

"In addition to other optional units, 'six units are prescribed for graduation from an accredited high school; three units of English, two units of Social Science, including one-half unit of Constitution of the United States, and one unit of Mathematics or a laboratory science'."—W. T. Markham, State Superintendent.

KENTUCKY

"In this state we require one unit of Algebra and one unit of Plane Geometry for high school graduation."—Mark Goodman, Public School Supervisor.

LOUISIANA

"Mathematics as a required subject for graduation has not been eliminated from the high school course of study. One unit of algebra is required for graduation, but students are urged to take more algebra than that."—Chas. F. Trudeau, State High School Supervisor.

MAINE

"In many of our high schools the mathematics requirement has been eliminated in the general curriculum. It is still required, however, in our college preparatory work."—Harrison C. Lyseth, Agent for Secondary Education.

MARYLAND

"No."—Albert S. Cook, State Superintendent of Schools.

MASSACHUSETTS

"Massachusetts has no state regulations with regard to the requirement of any high school subject except American History and Civics. Each community sets up its own graduation requirements. I think many of our high schools no longer require mathematics for graduation . . . "—Jerome Burtt, Supervisor of Secondary Education.

MINNESOTA

"Mathematics has not been required for some time."—E. M. Phillips, Commissioner of Education.

MISSISSIPPI

"Yes, in so far as State law and accrediting standards are concerned. However, most local school boards require one unit or more."—S. B. Hawthorn, State High School Supervisor.

MISSOURI

"This state requires one unit of Mathematics for graduation from high school."—J. R. Scarborough, Director of High School Supervision.

MONTANA

"No."—Elizabeth Ireland, State Superintendent.

NEBRASKA

"We are pleased to state that mathematics is still a required admission subject and that students seeking admission to the University of Nebraska are expected to have at least two years of high school mathematics taken in grades nine to twelve."—G. W. Rosenlof, Director of Secondary Education and Teacher Training.

NEVADA

"No."—Amy Hanson, Office Deputy.

NEW HAMPSHIRE

"Mathematics is a required subject in some of our high schools, it being required by the local boards. English and United States History

are the only subjects required by the State Board of Education. I would say that mathematics is gradually being eliminated as a required subject in many of the high schools. It still is and will continue to be required in the academic curriculum in most schools."—Russell H. Leavitt, High School Agent.

NEW JERSEY

"Mathematics has not been eliminated from the list of requirements for graduation in the high schools of New Jersey."—Howard Dare White, Ass't Commissioner of Education.

NEW MEXICO

"I wish to state that mathematics is still a required subject for graduation."—Mrs. Marianne Geyer, High School Supervisor.

NEW YORK

"Every secondary school of middle or high school grade must include in its curriculum English in each year, civics in the first or second year, physical education in each year, physiology and hygiene in the first year, one year of mathematics and American History in the third or fourth year."—Geo. M. Wiley, Ass't Commissioner.

NORTH CAROLINA

"No."—J. H. Highsmith, Director of Instructional Service.

NORTH DAKOTA

"Mathematics is not one of the state requirements in North Dakota for graduation from high school. The elimination was made in 1931."—John A. Page, Director of Secondary Education.

OKLAHOMA

"One year of mathematics is still required for graduation from the high schools of Oklahoma. This one year of mathematics may consist of a year of general or applied mathematics or a year of arithmetic, provided the course in arithmetic does not consist merely in a review of the work which should have been covered in the sixth, seventh and eighth grades."—J. Andrew Holley, Chief High School Inspector.

OHIO

"Mathematics is a recommended required subject."—L. W. Reese, High School Supervisor.

OREGON

"No."—C. A. Howard, State Superintendent of Public Instruction.

PENNSYLVANIA

"Mathematics as a required subject for graduation from the high schools of this State has not been eliminated. The minimum requirement as listed in our Standards for the Classification of Secondary schools is one unit in mathematics."—Walter E. Hess, Advisor, Secondary Education.

RHODE ISLAND

"No state courses of study are prescribed in Rhode Island. Hence, it has not been accurate to say that mathematics has been required as a matter of fact and law. Unfortunately, perhaps, the question of graduation from high school has not been distinguished from the question of admission to college. We know of no reason why a person could not be graduated from high school without mathematics, but so far as we know, all colleges require mathematics for entrance. Under our regulations, a person who completed unit requirements to total 14 point credits could be graduated without mathematics."—Walter E. Ranger, Commissioner of Education.

SOUTH CAROLINA

"Mathematics is not a required subject for graduation from accredited high schools; however, the programs of most high schools are so limited that it is necessary for pupils in these schools to take some mathematics in order to accumulate the required sixteen units."—Jno. G. Kelly, State High School Supervisor.

SOUTH DAKOTA

"Algebra or general mathematics is a required subject for graduation. Plane geometry was eliminated as a required subject in 1931-32."—R. W. Kraushaar, High School Supervisor.

TENNESSEE

"Three units in mathematics are now required for graduation from Tennesese high schools."—R. W. Vance, Supervisor.

TEXAS

"In my opinion the first two of the three questions should be answered in the negative."—M. B. Brown, Deputy State Superintendent.

UTAH

"Mathematics is still a required subject for graduation and it is also a required entrance subject for university and college admission. There are a number of leaders in Utah who would prefer to have mathematics (Algebra and Geometry) stricken from the required list for both

high school graduation and college entrance."—Chas. A. Skidmore, State Superintendent of Public Instruction.

VERMONT

"Am glad to send you information concerning the status of Mathematics. This subject is now a required subject in the high school curriculum under section 1277 of the general laws."—Charlotte M. Lowe, Secretary to the Commissioner.

VIRGINIA

"Mathematics as a required subject for graduation in the high schools of Virginia has not been eliminated. The State Board requires 2 units except for pupils in Vocational courses."—C. Y. Hyslup, Acting Supervisor of Secondary Education.

WASHINGTON

"One year of mathematics is required of all high school students."—N. D. Showalter, State Superintendent of Public Instruction.

WEST VIRGINIA

"One unit of mathematics is required of all pupils graduating from a first class high school in West Virginia at this time."—A. J. Gibson, State Supervisor of High Schools.

WISCONSIN

"Mathematics is no longer a required subject for graduation from high schools in this state . . . . The college and university presidents of Wisconsin have recently agreed to eliminate it as a requirement for college entrance in the state. Our department is encouraging schools to make both algebra and geometry elective and to transfer them to the later years of the high school course to be carried only by those pupils who decide that they will need them for some particular purpose."—J. T. Giles, State High School Supervisor.

WYOMING

"While mathematics is not required of every individual graduating from high school it is taken by nearly all who do graduate. The State Department of Education requires that a certain curriculum be offered in every accredited high school. This must include at least two years of mathematics."—B. H. McIntosh, Commissioner of Education.

DISTRICT OF COLUMBIA

"One year of mathematics is required but this year it may be commercial arithmetic rather than algebra, if the pupil so elects."—S. E. Kramer, First Assistant Superintendent.

Correlation and Secondary School Mathematics

By W. D. Reeve
Teachers College, Columbia University

Anyone who follows the trend of the times and who reads widely cannot fail to appreciate the fact that the static condition of the curriculum is responsible for our failure to make reasonable progress in reorganizing our secondary schools. This static condition is due in large measure to the tendency of teachers to cling to the traditional program and also to the fact that they have become so enamored of the subjects they teach that they are not always conscious of what is going on around them. Moreover, they do not appreciate the full significance of recent criticisms of the unsatisfactory situation in our secondary schools.

In this respect mathematics and particularly algebra and geometry are coming in for an appreciable share of criticism. Everybody from the general educationist to the man on the street feels himself competent to criticise. In three recent plays in New York City one of the characters in each play takes great satisfaction in taking a rap at mathematics. One character remarked, "Why she is as crazy as my geometry teacher," implying of course more than probably was intended. The interesting thing about each incident is that when these attacks were made the audience emitted a groan of approval.

When one group of mathematics teachers was recently reminded of the numerous attacks on the teaching of algebra one teacher replied "We should worry, we are no worse than the teachers of English!" Perhaps not. The teaching of English has frequently been bad enough. However, such an attitude should not be consoling to progressive teachers.

While most of the criticisms of mathematics are hurled at a type of mathematics teaching that has been outmoded in many schools, there is still just cause for complaint. Why is it that so many people who have studied algebra or geometry have such a terrible complex against the subject? What is the real reason?

A careful study of the situation in mathematics does not indicate that the teachers have failed any worse than the bankers, the captains of industry, or even the school superintendents in recent years. However, we know that there is room for improvement all around.

The trouble is not so much with mathematics as one of the great branches of learning, but with the stupid way in which it is so often presented to the pupils, even in arithmetic.

As one of the great fields of knowledge mathematics is important in the education of every American citizen, but we must decide just what parts of mathematics will be most valuable. Moreover, in this complex civilization which we are now entering a knoweldge of many things in mathematics is becoming increasingly important and teachers must keep up with the procession if they are to be leaders of the next generation.

It is obvious that a person cannot teach what he does not know and many of our teachers are not scholarly enough. Besides, they know too little outside their own "specialty" to make mathematics properly serve the other great fields of knowledge. What we need more than anything else right now is better trained teachers—teacher-scholars as Professor Bagley calls them, who know not only the subjects they have to teach but who are also more generally cultured than many of our teachers of formal algebra. We have an oversupply of teachers at present, but we do not have an oversupply of the right kind of teachers of mathematics. The day is past when anyone should be permitted to teach algebra merely because he is trained to be an athletic coach. If our teachers were required even to approximate the standards for teaching in Germany, we would soon see the dawn of a new day in the teaching of mathematics.

In the Hall of Science at the Century of Progress Exhibition in Chicago last year there was represented on one of the walls "The Tree of Knowledge." At the base of this tree stood mathematics supporting the other great branches like the applied sciences, and subjects like education on the higher branches. While some people may question that mathematics is so important, it is not surprising to anyone who goes to the root of the matter. Mathematics has contributions to make to all of these other fields like biology, astronomy, physics, chemistry, engineering. The problem for the teacher is to equip himself as well as he can in all of these fields so that he can bring out the proper correlation when the time comes. And it ought to be emphasized that the mathematics teacher, other things being equal, is the best qualified to say when and where mathematics will be most helpful. However, any correlation of subjects in the schools should afford an opportunity for the right kind of cooperation between the various fields, and if the right kind of correlation is made, a great deal of time can be saved.

There is nothing new in the idea of correlation. It is a practice which good teachers have always believed worth while. However, it must not be done without care and thoughtful planning. Just as mathematics can make important contributions to physics, so physics can be used to clarify and enrich mathematics. In reality it is not a question

of either, but a question of precedence and emphasis if we are to give our pupils a well rounded education.

It ought to be said here that before teachers can correlate properly mathematics with other fields, they ought to learn how to correlate the various parts of mathematics. They should first learn how and where arithmetic and informal geometry can be correlated, how and where algebra may be best correlated with arithmetic and informal geometry, and so on. Unless we can do this there is small chance that we can successfully correlate mathematics with music, the arts, and other applied fields.

Whether we like it or not, an attempt is being made to correlate the various subjects in the curriculum. While this is at present confined largely to the elementary school, there is a strong tendency to do the same thing on the secondary level. The idea at best is sound. We teachers of mathematics should direct and not block such a movement. We need to show how the study of mathematics will help one to be a better student of science, music, or the arts. We need also to teach mathematics as a method of thinking so that the pupils who sit at our feet will go out into life able to think better because they have learned to think in studying their mathematics.

Someone may say that the task of correlation is so great as to be impossible of realization. This need not be true if we begin on a small scale and try each year to educate ourselves better in each field.

Some General Remarks Concerning Scientific Exposition

By Rudolph E. Langer
University of Wisconsin

It may seem strange to you that on this occasion of a gathering of mathematicians I should have chosen to speak upon a subject in no way peculiarly associated with mathematics. Let me hasten, therefore, to disclaim it as a matter of choice, for when your Program Committee honored me with the invitation to be among your speakers, a stipulation of the topic was included, and the matter seemed quite without alternative.

*Address delivered by invitation of the program committee before the Wisconsin Section of The Mathematical Association of America at Beloit, Wisconsin, April 8, 1933.

I will confess that I have faced the prospect only with misgivings. The preparation of an exposition has never been for me an entirely easy task, and to compound one upon a subject, in which as a student I won no noticeable laurels, promised to be doubly difficult. As a teacher I have, to be sure, found opportunity at times to assist in the expository efforts of my students by suggestion and criticism, but how different that to the role I now find myself in standing here before you. You may believe that thoughts of subterfuge have not failed to present themselves. There was appeal in the thought that a feint or two at expository matters might be made to shield a retreat into some other line of discourse in which I might perhaps with less tenuous expectations hope to merit your attention. I remember reading once that Ruskin, on the occasion of a scheduled lecture on crystallography, actually spoke upon architecture, and subsequently assured an inquisitor that even if he had not forgotten to begin upon crystals, he would at all events soon have found himself upon the architectural theme.

As you will see, however, I have put temptation behind me, and this mainly because, so far as the subject itself is concerned, I can think of none more immediately worthy of discussion. And so I will launch forth upon it, hoping all the while to conceal as well as I may my own incompetence for the task by avoiding so far as possible all those matters and issues which lie more specifically within the province of the man of letters. Should I not entirely succeed in this, I will console myself as did Mathew Arnold once on finding himself required to speak on a subject of science, with the thought that my shortcomings will be so abundantly evident that no one will be in the least danger of being misled.

> "Somewhere in the oldest English writings," says Barret Wendell in his *Literary History of America,* "there is an allegory which has never faded." "Of, a night," it tells us, "a little group was gathered about the fireside in a hall where the flicker of flame cast light on some and threw others into shadow, but none into shadow so deep as the darkness without. And into the window from the midst of the night flew a swallow lured by the light: but unable by reason of his wildness to linger among men, he sped across the hall and so out again into the dark, and was seen no more."
>
> "To this day," . . . he continues, "the swallow's flight remains an image of earthly life. From whence we know not, we come into the wavering light and gusty warmth of this world: but here the law of our being forbids that we remain. A little we may see, fancying that we understand—the hall, the lords and the servants, the

chimney and the feast; more we may feel—the light and the warmth, the safty and the danger, the hope and the dread. Then we must forth again, into the voiceless unseen eternities.

"But the fleeting moments of life, like the swallow's flight, are not quite voiceless; as surely as he may twitter in the ears of men, so men themselves may give sign to one another of what they think they know, and of what they know they feel. More too: men have learned to record these signs, so that long after they have departed, others may guess what their life meant. These records are often set forth in terms which may be used only by those of rarely special gift and training—but oftener and more freely they are phrased in terms which all men learn somehow to use—the terms of language."

In this passage a great master of language has recorded his thought, in a manner beautiful, and in terms forceful and clear. In the beauty of its expression the composition is the creation of an artist; in the clarity and effectiveness of its terms, it is the work of a superb craftsman—of one who knew how with his tools of language to hew out the embodiment of his ideas. Even though I would, I could not speak to you of the use of language as an art. To craftsmanship, however, even the ungifted may attain, and so I shall try to speak to you on the use of language as a tool.

We all, I suppose, like to think of ourselves as having devoted our lives to the great cause of the advancement of science and the effective dissemination of scientific knowledge. In this immense cooperative enterprise it is given to but few to fill the role of discoverers of significant facts and principles, and to those so endowed we are all eager to give appropriate homage and honor. But there is work for great numbers in posts less comspicuous though still important and these are the posts to which most of us aspire. New theories and facts must be brought into the closest possible relations with older ones. The implications and reactions of different theories and facts upon each other must be probed, and in the light of the relations revealed reorganizations and simplifications in all parts of the science must be worked upon continually, in order that penetration into the structure itself may become progressively easier, and that the top men may be hampered as little as possible by the lesser and more easily removable obstructions.

Now in this work of organization, consolidation, dissemination and diffusion, of scientific fact and theory, the indispensable tool is language. Explanation and elucidation are necessary at every turn, and whether we wish it or not we are continually engaged in exposition. To the

average man of science, therefore, no phase of his activity is of greater importance or more worthy of his attention. Mutually dependent upon one another as we are, the degree of consequence we attain for our work hinges in a very real measure upon the degree of our skill in exposition. To neglect the cultivation of this skill inexorably means to divest oneself of a definite modicum of effectiveness.

With all of us, by far the greater portion of our expository activity is of the extemporary type. In conversation, in class-room discussion, in reply to questions, we plunge impromptu into explanations. Such expositions are almost invariably oral, and, though subject to definite principles, rarely involve consciousness in the application of them. Less frequent but perhaps of greater immediate moment are the occasions upon which we engage in exposition with deliberation and with a well defined purpose. These are the occasions I have particularly in mind. Be such an exposition one of the spoken or of the written word, be it scientifically of an elementary or of an abstruse character, be it addressed to students of meagre training or to a gathering of erudite scholars, the principles are the same, and the expositor will invariably do well to observe the rules.

Perhaps the first requirement of a successful exposition is an adequate, or better, a superabundant knowledge of the subject. To attempt an explanation of what we ourselves do not thoroughly understand is surely futile; nor is an incidental effort to conceal our shortcomings apt to be very successful. Like the proverbial voice of conscience, our uncertainties are all too likely to accompany us whichever way we turn. They must be laid low to hamper the exposition no more. Perhaps I cannot do better toward emphasizing this point than to quote to you from an essay by a certain notably successful teacher and writer, Professor G. H. Palmer of Harvard University. He says:

> "In preparing a lecture, I find I always have to work hardest on the things I do not say. The things I am sure to say . . . I find, are not enough. I have a broad background of knowledge which does not appear. I have to go over my subject and see how the things I am to say look in their varous relationships, tracing out connections which I shall not present. One might ask 'What is the use of this? Why prepare more matter than can be used?' The answer is 'I cannot teach right up to the edge of my knowledge without a fear of falling off. My auditors discover this fear and my words are ineffective. They feel the influence of what I do not say. One cannot precisely explain it—but when I move freely across my subject as if it mattered little on what part of it I rest—they get a

sense of assured power which is compulsive and fructifying, and the subject acquires consequence'."

The principle here involved is, of course, familiar to all of us. We constantly apply it in expecting the prospective teacher to submit to a degree of training which transcends by far the particular subjects which are ultimately to be taught. Whether the subject to be expounded be a complicated or a simple one, the expositor will find it essential to delve into it, to assure himself of a complete understanding of it, and, when possible, to attain a conviction that even the less obvious implications of it are no mysteries to him. This point will bear no slighting if one wishes to be really qualified for the role of an expositor.

Let me hasten to correct an impression which I may have given you that one should not venture even upon the preliminaries of an exposition until perfection in the mastery of subject has been attained. Quite the contrary is often the case. The attempt at exposition is in itself perhaps the most effectual single means for attaining a truly dominant mastery of subject. The exigencies of careful verbal formulation serve so powerfully toward clarifying the thought itself that it is a familiar saying that "Thoughts are as dependent upon words as words are upon thoughts." All too often our disorganized reasoning is apt to be scatterbrained more or less. If so, there is no procedure more effective toward producing a logical soundness and a clarity of understanding than that of ordering the facts in the form of a written record. It is believed by many that the recession of the Chinese race from their early position of great cultural eminence may be ascribed very largely to the cumbrousness of their language. On the other hand, it is amply evident that the peoples which are scientifically preeminent in the present are also those which admittedly possess the languages which afford the maximum power of expression and the greatest adaptability to shades of meaning. We ourselves have in this respect no grounds for complaint. It depends only upon our own will whether or not we train ourselves to use worthily the wonderful linguistic instrument which is our birthright.

A noted teacher once remarked that "We speak our first thoughts but should write only our second, or better still, our fourth." This brings me to the second point which I wish to raise, namely, that of organization. In impromptu oral exposition the organization is all too often in the nature of things loose or even absent. In a premeditated or planned work, however, whether it be oral or written, a lack of organization is never to be condoned. The problems of organization are seldom simple, rather they often tax the expositor's skill and his

patience and endurance as well. He who would expound successfully must always seek to seize upon the central ideas and facts to be set forth, and keeping these all the while in mind, must marshall the subsidiary facts and considerations in an orderly manner about them. The precise way in which this is to be done will, of course, be influenced in any given instance by many circumstances. These the expositor must bear constantly in mind. I mention by way of illustration such features as the character of the subject, whether long or short, simple or abstruse, etc.; the prospective audience, a term I use to designate either auditors or readers as the case may be; the mission or purpose of the exposition; the exigencies of presentation or publication; and other such like. All such considerations do and should play a part as influences upon the systematic arrangement of a presentation.

If the work is a long one, it will be necessary to organize carefully not merely the whole but its parts as well. While working in the rough the expositor must seek to visualize the finished whole, and when an abundance of difficulties is inherent in the subject itself, as is the case with mathematics, he must plan to develop his chain of reasoning so that the facts to be uncovered are displayed in a completely ordered procession. Different minds are differently equipped and constituted, and no expositor has the right to assume on the part of any audience the same degree of familiarity with the facts or their interrelations as he himself possesses. If he be endowed with imagination, therefore, he will pause frequently to regard himself in his reader's place, and from the point of view he so obtains, will seek out the line of advance which offers the least resistance to the understanding.

It has sometimes been held up as an ideal for scientific theories that they should be considered as finished only when they may be made intelligible in simple terms "to the man in the street." As a goal such would surely be but rarely attainable. As an ideal or as a challenge, it leaves little to be desired. Expository writing is by its very nature bent toward a single mission, to make its subject clear in the simplest terms; not necessarily to the man in the street, but to the audience for which it is intended. If, to be concrete, I present an isolated subject fragment to a group of students with general but no specialized training, shall I say at a meeting of a Mathematics Club such as we all know them, then I may use general technical terms but avoid those of the specialty, and while I may omit detail if the point at issue is one of general principle, I must take care when the special features are drawn in that I supply to my hearers a grounding which may extend even to the very elements. My mission is then not to make the subject seem deeply complicated or profound, not to impress my hearers with my

own learning such as it may be, but simply and solely to keep within my guidance the understanding of the audience. If I guard against trivialities I need scarcely fear to make the subject too simple. Should the occasion, on the other hand, be a scientific symposium for specialists such as the great mathematical societies at times arrange, then the interpretation of the phrase "in the simplest terms" might well be appropriately revised. Surely I need not labor this point further, nor discourse longer on its bearing upon the organization of an expository work.

(To be continued in the next issue)

Two Graphical Treatments of the Moment of Inertia of a Plane Lamina

D. L. HOLL
Iowa State College
Ames, Iowa

Consider a plane lamina S in the x-y plane, and define the moments of inertia and 'product of inertia'* of the lamina for an orthogonal set of axes in the plane of S by

$$I_x = \int y^2\, dA\ ,\ I_y = \int x^2\, dA\ ,\ I_{xy} = -\int xy\, dA\ ; \qquad . \ . \ (1)$$

where dA is an element of the area, and the integration extends over the entire lamina S.

For an orthogonal set of axes x'-y' rotated through an angle θ, one finds, using the orthogonal transformation

$$\begin{cases} x' = x\cos\theta + y\sin\theta \\ y' = -x\sin\theta + y\cos\theta \end{cases}, \qquad . \ . \ . \ (2)$$

the new components of inertia to be

$$I_{x'} = (I_x + I_y)/2 + \frac{(I_x - I_y)}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{y'} = (I_x + I_y)/2 - \frac{(I_x - I_y)}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

*In this paper the three quantities will be referred to as components of the inertia tensor. The taking of the negative integral as the product of inertia turns out to be more natural in vector analysis. See C. E. Weatherburm, Advanced Vector Analysis, p. 104.

$$I_{x'y'} = I_{xy} \cos 2\theta - \frac{(I_x - I_y)}{2} \sin 2\theta \qquad . \quad (3)$$

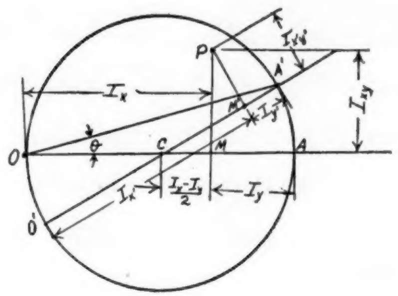

fig 1.

Referring to Fig. 1, one finds a graphical representation of these new components of inertia. Construct a circle of diameter

$$OA = I_x + I_y \text{ with } OM = I_x \text{ and } MA = I_y.$$

At M erect an ordinate MP of the same sign and magnitude as I_{xy}. Corresponding to any rotated set of axes, there is constructed the angle $AOA' = \theta$, and the central angle $ACA' = 2\theta$. Projecting the point P upon the diameter $O'CA'$, we find the components of (3) to be $O'M'$, $M'A'$, and $M'P$ respectively.

From (3), the condition for the vanishing of $I_{x'y'}$ is also the condition for $I_{x'}$ to be a maximum and $I_{y'}$ to be a minimum. The condition is

$$\tan 2\theta_1 = 2I_{xy}/(I_x - I_y) \qquad . \quad . \quad (4)$$

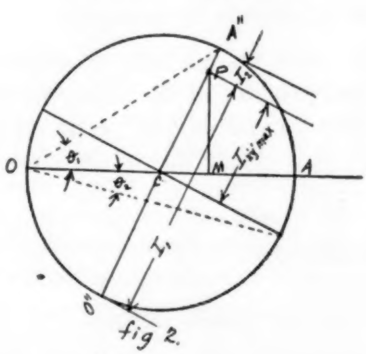

fig 2.

Geometrically this is equivalent to constructing the angle $ACP = 2\theta_1$ and the directions *for principal axes of inertia* of the lamina S for the

point 0, which is still taken as the origin, are in the direction of OA″ and perpendicular to OA″. From Fig. 2 one readily obtains the maximum and minimum values of the components (for all orthogonal axes intersecting at point 0) to be

$$I_1 = I_{max} = (I_x + I_y)/2 + \tfrac{1}{2}\sqrt{(I_x - I_y)^2 + 4I^2_{xy}},$$

$$I_2 = I_{min} = (I_x + I_y)/2 - \tfrac{1}{2}\sqrt{(I_x - I_y)^2 + 4I^2_{xy}}. \qquad . \; . \; (5)$$

The condition for $I_{x'y'}$ to become a maximum is

$$\tan 2\theta_2 = -(I_x - I_y)/(2I_{xy}) \qquad . \; . \; (6)$$

From Fig. 2, one observes that $\theta_2 = \theta_1 \pm 45°$. Also when $I_{x'y'}$ is a maximum, there results

$$CP = (I_{x'y'})_{max} = \tfrac{1}{2}(I_1 - I_2) = \tfrac{1}{2}\sqrt{(I_x - I_y)^2 + 4I^2_{xy}},$$

$$[I_{x'}]_{\theta_2} = [I_{y'}]_{\theta_2 + 90°} = \tfrac{1}{2}(I_x + I_y) \qquad . \; . \; (7)$$

From geometrical considerations it is not difficult to verify the following invariants:

$$I_x + I_y = I_{x'} + I_{y'} = I_1 + I_2 = \text{a constant}, \qquad (8)$$

$$\begin{vmatrix} I_x & I_{xy} \\ I_{xy} & I_y \end{vmatrix} = \begin{vmatrix} I_{x'} & I_{x'y'} \\ I_{x'y'} & I_{y'} \end{vmatrix} = \begin{vmatrix} I_1 & 0 \\ 0 & I_2 \end{vmatrix} = \text{a constant} \;. \qquad (9)$$

Obviously these are the invariants of the inertia tensor

$$\begin{pmatrix} I_x & I_{xy} \\ I_{xy} & I_y \end{pmatrix}$$

and are expressible in the simple theorems:

A. The diameter of a circle is constant;

B. If from any point in a circle, a perpendicular is dropped upon any diameter, the product of the segments of the diameter thus intercepted exceeds the square of the perpendicular by a constant, which is $R^2 - r^2$ where R is the radius of the circle and r is the radius vector of the chosen point.

Another Representation.

From equations (2) one readily obtains

$$x' + iy' = (x + iy)(\cos\theta - i\sin\theta) = (x + iy)e^{-i\theta},$$

and $$y'^2 - 2ix'y' - x'^2 = (y^2 - 2ixy - x^2)e^{-2i\theta},$$

$$\text{or} \qquad (I_{x'}-I_{y'})/2+i\,I_{x'y'}=\left(\frac{I_x-I_y}{2}+i\,I_{xy}\right)e^{-2i\theta}. \tag{10}$$

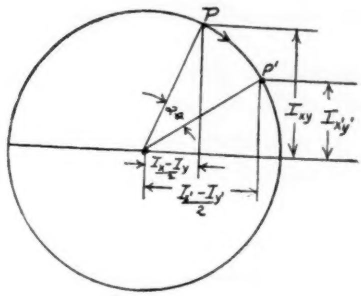

fig 3.

This equation shows that the vector representing the left member of (10) may be obtained from the vector of the right member by a clockwise rotation of 2θ, as is shown in Fig. 3. By addition and subtraction of (8) and (10) and equating reals and imaginaries, one obtains (3).

In the complex plane of Fig. 3, it is seen that the radius vector has a modulus $\frac{1}{2}(I_1-I_2)$ where the subscripts denote the maximum and minimum values defined by (5). Hence the maximum value of $I_{x'y'}$ must exist when $I_x=I_y$ and has a magnitude equal to the radius and is identical with the first of equation (7).

Thus far the origin 0 has been an arbitrary point in the plane of the lamina S. In case the origin coincides with the centroid of S and a principal axis is determined in the direction θ_1, then if a line through the centroid is a principal axis at one of its points, by the parallel axis theorem it is a principal axis at all its points. Also each line not through the centroid is a principal axis at one of its points.

Additional theorems on the loci of the principal directions leading to a set of confocal ellipses and hyperbolas referred to the foci of inertia (Murnaghan, Theoretical Mechanics p. 193) may be inductively built up from these geometric considerations.

The writer has found that students of Elementary Analytical Mechanics find the above treatments of moments of inertia to produce quantitative results and in some aspects clearer concepts than by the more usual method of the ellipse of inertia.

The Teachers Department

Edited by

Joseph Seidlin and W. Paul Webber

My first official article for the new department assumes the clarion call quality. The task of improving teaching is not a one-man job. I believe it is a case of "All good men must come to the aid of their party." It is hoped that we may succeed in spreading the gospel of these "good" men.

I am not an authority on the teaching of mathematics. I do not know that any one is. I do know, however, that teachers of mathematics cannot keep on teaching as they have been teaching without irreparable injury to their victims, their subject, and, ultimately, themselves. I have in mind the formidably rank rank and file of our frozen profession.

Our first task is to arouse the interest and secure the co-operation of our colleagues in all institutions offering courses in college mathematics. It is true, albeit a pathetic and painful truth, that all too many teachers of college mathematics are disinterested in the teaching of college mathematics. Of these, many claim that qualified teaching is a myth, that good teaching cannot be distinguished from poor teaching; others concede shades of differences in the qualities of teaching but regard the whole matter of teaching as relatively insignificant; still others recognize excellent teaching purely as a gift of the gods and ineffective teaching as a curse of the self-same gods, so that it is best for mere mortal man not to meddle with teaching as he finds it. I fear that for the present we must plan the campaign for improved teaching without the assistance and despite the prejudice of the disinterested teacher.

The first, and pressing, problem facing all teachers of mathematics concerned with the education of our youth is to arrive at some reasonable agreement as to the place and value of elementary college mathematics in a college curriculum. Presumably, teachers of mathematics feel that mathematics has certain educative values peculiarly its own in kind or degree, that mathematics contributes to better living, to a finer sense of citizenship, to more completely rounded personality in mathematically unique fashion. Certainly, teachers of mathematics must be willing and ready to meet the thesis that eliminating mathematics from college curricula will not affect adversely the education of youth in an enlightened democracy.

So long as "the doctors disagree" there is little hope of counteracting effectively the anti-educational forces now so actively and successfully engaged in opposing education in general and the educative values of mathematics, in particular. It is not primarily a matter of salvaging jobs for teachers of college mathematics, though that may color our views somewhat; it is rather a re-valuation of the usefulness, and place, and value of the teacher of college mathematics in the educational schemes of the nation. Teachers of college mathematics should not blind themselves to the fact that if they, themselves, fail to evaluate themselves and their work, others, less fit, less "scientifically-minded," less educationally-minded, but more prejudiced and embittered against "mathematics at large," will.

And, just between you and me and other teachers of mathematics, is it not likely that a careful, searching, and honest analysis of the courses we now teach and the way we teach them may bring about re-adjustments and consequent improvements which in time will result in more tangible proof of the values of mathematics than any number of pretty phrases tied to indefensible content and poor teaching?

And so, our first task is to arouse the interest of teachers of college mathematics in the fate of elementary college mathematics with regard to its position in the coming educational development of our youth. Can we accomplish this task through the department on the Teaching of College Mathematics in the *National Mathematics Magazine*? Let us try.

Yours, in a common cause,
JOSEPH SEIDLIN

Book Review Department

Edited by
P. K. SMITH

Differential Equations. By Max Morris and Orley E. Brown. New York, Prentice-Hall, Inc., 1933; vii + 409 pages.

The last few months have witnessed the publication of several new texts on differential equations as well as a new edition of Cohen's *Elementary Treatise on Differential Equations.* Happily all of them are quite individual in their treatment so that an instructor may choose for widely different courses. In the following there will be noted only some of the features which distinguish this text from the older ones

covering the classical material of ordinary and partial differential equations.

In Chapter I a statement (without proof) is made of the existence theorems for one first order differential equation and for a system of first order equations. Singular solutions are introduced in connection with the general solution before singular solutions are discussed in detail. Problems concerning the locus of maximum and minimums and inflection points of integral curves help to sharpen the concept of first order differential equations.

Chapter II contains the correct rule for solving exact equations (some classic texts avoid the issue). Homogeneous functions are defined in general (not under the restrictive hypothesis of polynomials in x and y). Riccati equations are not slighted. Many of the rules for finding integrating factors are relegated to the problems (with sufficient data and hints as to procedure). There is no separate résumé of methods of solving first order equations but the table of contents is sufficiently precise.

Chapter III discuses singular solutions rather than the loci of singularities of integral curves. Attention is called to the limiting positions of integral curves when the parameter (constant of integration) tends toward critical values or become infinite.

Chapter IV leads up to the discussion of D operators by a series of remarks on simple problems. The discusion is technically correct. Wronskians and linear independence of solutions are introduced early in the game.

Chapter V, devoted to the numerical approximation of solutions, is followed by a chapter on integration in series with a quite complete treatment of the second order linear equation and applications.

Chapters VII-X take up total differential equations, systems of total differential equations, Jacobi's multipliers, partial differential equations of the first order with methods of Lagrange, Charpit and Jacobi, operational method for linear partial differential equations with constant coefficients, Monge's method for partial differential equations of second order, and Laplace's transformation.

The wealth of problems is a decided feature of the book. The authors have not given references to a host of special texts but have chosen two, namely Wilson's *Advanced Calculus* and Dickson's *First Course in the Theory of Equations,* which should be available to every instructor and student. Determinants, matrices and functional determinants are used freely throughout.

A review to be complete should contain at least some criticism. The publishers have anticipated one item of criticism by sending to instructors using the text several mimeographed correction sheets. Many errors appear in the answers to the problems and a few errors in the statements. The geometric point of view has been neglected somewhat in behalf of the analytic point of view. The reviewer has enjoyed using this text in a course on ordinary differential equations.

W. E. BYRNE.
March 19, 1934.

Notes and News Department

Edited by
I. MAIZLISH

AN APPEAL

This Department of NOTES AND NEWS has been added to the *National Mathematics Magazine* because of the conviction that it will add to the "human" side of the Magazine, and also in the sincere hope that it will enhance your interest in it. To accomplish this objective, it is absolutely necessary that all the readers of the Magazine cooperate with the Editor. An appeal is therefore made to each of you at this time to send us ALL ITEMS of interest to mathematicians, and academic people in general—local, national, and international—such as promotions, honors, scientific news of universal appeal, notes of addresses delivered by scientific people, societies and clubs, scientific and engineering achievements, and other items of a similar nature. Please send all such news and notes to I. Maizlish, Centenary College, Shreveport, Louisiana. There is no other way by which the Editor of this Department can secure this material. Of course, we want the "Notes and News" items to be "first hand," but the following are a few such items taken at random from one or two well-known journals, to illustrate the type of material wanted for this Department. No matter where or how you get the "Notes and News" items, if they interest YOU, please be sure to send them to the Editor. With your cooperation, this Department will endeavor to add to the moments of pleasure you spend in reading the *National Mathematics Magazine*.

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The Abbe Georges Lemaitre, professor of astrophysics at the University of Louvain, will be visiting professor at the Catholic University of America in Washington, D. C. He will lecture on the astronomical applications of the theory of relativity and will conduct a seminar

for advanced students of physics and mathematics in the graduate school of that institution.

The new science building at Radcliffe College has been named William Elwood Byerly Hall in honor of W. E. Byerly, Perkins professor of mathematics emeritus at Harvard University, who served for more than thirty years as Chairman of the Academic Board at Radcliffe.

Among the American mathematicians who have been elected fellows of the Econometric Society are the following: G. C. Evans, Harold Hotelling, C. F. Roos, and E. B. Wilson.

Dr. George David Birkhoff, professor of mathematics at Harvard University has been awarded a prize of ten thousand lire ($825) donated by Pope Pius XI in an international competition for the best book on "Systems for the Solution of Differential Equations." The award was made during exercises inaugurating Pontifical Hall of Science at Vatican City, on December 17, 1933.

In the mathematics section of the Basic Science exhibits at the Century of Progress Fair are certain models of projections of hyper-space regular figures made by P. S. Donchian, of Hartford, Conn. The set includes the six regular figures of 4-dimensional space and the hypercube series extended to n-dimensions.

Professor P. A. M. Dirac, of the University of Cambridge, will be Visiting Professor of Mathematical Physics at the Institute for Advanced Study, Princeton, New Jersey, for the academic year, 1934-35.

Dr. Irving Langmuir, Associate Director of the Research Laboratory of the General Electric Company, has been appointed honorary Chancellor of Union College for the present academic year.

Dr. E. B. Wilson, Professor of Vital Statistics in the School of Public Health, Harvard University, delivered the principal address in connection with the annual Sigma Xi day held at the University of Rochester.

Oglethorpe University has conferred an honorary doctorate on Professor Archibald Henderson of the University of North Carolina.

Dr. L. P. Eisenhart, Professor of Mathematics and dean of the faculty at Princeton University, has been elected dean of the graduate school.

Honorary degrees conferred at the commencement of Washington and Jefferson College included the degree of LL. D. on Henry A. Wallace, Secretary of Agriculture, and the degree of doctor of science on Dr. A. H. Logan, dean of the Mayo Clinic at Rochester, Minnesota.

Assistant Professor K. W. Lamson, of Lehigh University, has been promoted to an associate professorship of mathematics.

Problem Department

Edited by

T. A. BICKERSTAFF

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of most interest to the readers, it is believed that regular text-book problems are, as a rule, less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

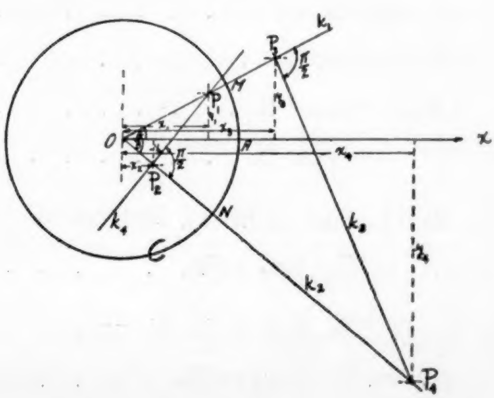
SOLUTIONS

No. 43. Proposed by C. D. Smith, Mississippi State College.

Given a quadrilateral $P_1P_2P_3P_4$ with the usual rectangular coordinates. Find the conditions under which

$$x_1x_3 - x_2x_4 + y_1y_3 - y_2y_4 = 0$$

The following solution is offered by A. W. Randall, Prairie View State College, Prairie View, Texas:



Let the quadrilateral be determined by the points P_1, P_3, P_4, P_2. Draw the lines k_1, k_2 through the point-pairs P_1, P_3; and P_2, P_4 respectively. Let k_1, k_2 intersect in 0. With any radius OA describe the circle C, cutting k_1, k_2 in the points M, N. Draw k_3, k_4 perpendicular to k_1, k_2 respectively, as shown in the figure. Now points P_1, P_3, P_4, P_2 are concyclic. Hence

$$OP_1 \, . \, OP_3 = OP_2 \, . \, OP_4 = \overline{ON}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

From (1), we see that k_4 is the polar of P_4. Since k_4 intersects k_1 in P_1, it is obvious that k_3 is the polar of P_1. Hence it is clear that the sides $P_3 P_4$, $P_1 P_2$ of the given quadrilateral must be the external and internal polars with respect to the circle C.

Let angles

$$AOP_1 = G,$$

and

$$AOP_2 = B.$$

From the figure,

$$OP_1 = x_1 \sec G, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$OP_3 = x_3 \sec G, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$OP_2 = x_2 \sec B, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$OP_4 = x_4 \sec B. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Again,

$$OP_1 = y_1 \csc G, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$OP_3 = y_3 \csc G, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$OP_2 = -y_2 \csc B, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$OP_4 = -y_4 \csc B. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Substituting, (2), (3), (4), and (5) in (1), we have

$$x_1 x_3 \sec^2 G = x_2 x_4 \sec^2 B. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

And again using (6), (7), (8), and (9) in (1), we get

$$y_1 y_3 \csc^2 G = y_2 y_4 \csc^2 B. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

From the conditions of the problem, x_1x_3 must be equal to x_2x_4. From (10) it is clearly seen that this condition is satisfied when, and only when.

$$\angle G = \angle B.$$

Then (10) becomes

$$x_1x_3 - x_2x_4 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Similarly (11), reduces to

$$y_1y_3 - y_2y_4 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Adding (12) and (13), we obtain our desired equation

$$x_1x_3 - x_2x_4 + y_1y_3 - y_2y_4 = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Now we will summarize the conditions that must be imposed on the given quadrilateral to get (14):

(a) A pair of opposite angles must be right angles,

(b) The points P_1, P_2, P_3, P_4 must be concyclic,

(c) A pair of opposite sides must be external and internal polars with respect to a circle whose center is at the point of intersection of a pair of opposite sides, and

(d) The x-axis of the points must bisect the angle formed by the pair of sides intersecting at the center of the circle C.

No. 55. Proposed by Richard A. Miller, University of Mississippi.

To construct an equilateral triangle with vertices on three given parallel lines.

Construction by Lloyd C. Bagby, Linsly Institute of Technology Wheeling, W. Va.

Assume the three parallel lines, l_1, l_2, l_3. On l_1 select a point P_1. On l_2 select a point P'_2. Construct the equilateral triangle P_1, P'_2 x'_2. On l_2 select another point P_2''. Construct the equilateral triangle $P_1P_2''X_2''$. Join x'_2 and x''_2 and extend until intersection P_3 on l_3 is obtained. Construct the triangle $P_1P_3P_2$. P_2 will lie on l_2, $P_1P_2P_3$ is the desired triangle.

Also solved by Buford E. Gatewood, L. P. I., C. A. Balof, Director of Athletics, Lincoln College, Lincoln, Ill.; A. W. Randall, Prairie View, Texas; Vincent R. Noll, State Teachers College, Milwaukee, Wis., Exa Hardin, Prairie View, Texas; W. B. Clarke, San Jose, Cal., and a note was submitted by Earl Thomas, L. S. U.

No. 56. Proposed by Henry Schroeder, L. P. I.

A circle with radius a moves with its center on the circumference of an equal circle, and keeps parallel to a given plane which is perpendicular to the plane of the given circle.

Find the volume of the solid it will generate.

Solution by C. A. Balof, Lincoln College, Lincoln, Ill.

Let the fixed circle be the circle $x^2+y^2=a^2$, and let the generating circle move parallel to the xz plane. A cross-section of the solid generated, parallel to the xz plane, consists then of two intersecting circles. The area of this cross-section is readily determined to be

$$(1) \qquad 2\Pi a^2-2a^2 \text{ arc sin } 1/a(a^2-x^2)^{1/2}+2x(a^2-x^2)^{1/2}$$

and the volume of the solid is thus given by

$$(2) \qquad V=2\int_0^a \left\{ 2\Pi a^2-2a^2 \text{ arc sin } 1/a(a^2-x^2)^{1/2}+2x(a^2-x^2)^{1/2} dy\right.$$

where (3) $x^2+y^2=a^2$

and (4) $dy=-x(a^2-x^2)^{-1/2}dx$

Then (2) becomes

$$(5) \quad V=4\int \left\{ \Pi a^2x(a^2-x^2)^{-1/2}-a^2x(a^2-x^2)^{-1/2} \text{ arc sin } 1/a\,(a^2-x^2)^{1/2}+x^2 \right\} dx$$

from whence

$$(6) \quad V=4\left\{ -\Pi a^2(a^2-x^2)^{1/2}+a^2(a^2-x^2)^{1/2} \text{ arc sin } 1/a(a^2-x^2)^{1/2}+a^2x+{}^1/_3x^2 \right\}_0^a$$

(7) $V = 2\Pi a^3 + 16/3\ a^3 = 2/3\ a^3\ (3\Pi + 8)$

Also solved by Lloyd C. Bagby, Linsly Institute of Technology, Wheeling, W. Va., A. W. Randall, Prairie View, Texas, F. A. Rickey, L. S. U., and W. B. Clarke, San Jose, Cal.

PROBLEMS FOR SOLUTION

No. 61. Proposed by A. F. Moursund, University of Oregon, Eugene, Oregon.

Show that

$$\sum_{i=2}^{P+1} \frac{(-1)^i (P+1)!}{i(i-2)!\,(P+1-i)!} = 1.$$

No. 62. Proposed by A. F. Moursund, University of Oregon.

Given

(1) $q(t)$ is non-negative and monotone increasing on $(0,1)$:

(2) $$\int_0^1 q(t)dt = 1$$

Show that $$\int_0^1 \left.\begin{matrix}\sin \\ \cos\end{matrix}\right\} nst\ q(t)\ dt \not> Q(h)$$

Where

$$Q(h) = \int_{1-h}^{1} q(t)\ dt,\ h = \Pi/ns \not> 1$$

No. 63. Proposed by Walter B. Clarke, San Jose, California.

Line AB is one side of a triangle. Vertex C is to be located so that the projection on AB of HO equals that of IV. Point H is the orthocenter, O is the circumcenter, I is the incenter, and V is the concurrent point of three lines, each drawn from a vertex to the point half way around the perimeter of the triangle. What is the locus of C?

No. 64. Proposed by C. L. Wilson, Prairie View State Normal and Industrial College, Prairie View, Texas.

Given an oblique triangle ABC, with sides opposite, abc, b is the base, h is the altitude, and A (equal to θ) is a base angle. From A, a line is drawn cutting side a at O, forming a triangle AOC, whose base is b and whose altitude is x. If h/x equals r, find an expression for angle OAC in terms of h, x, θ, r, and b.

No. 65. Proposed by W. B. Clarke.

Considering only triangles whose sides are consecutive integers, and whose area is an integer, find the area of the triangle which is next larger than the one whose area is 16296.

LATE SOLUTIONS

No. 52 and 54 by C. A. Balof, and No. 54 by Richard A. Miller.